A New Look @ Parabolas (Section 3-11)

* Vertex Form: $y = a(x-h)^2 + k$

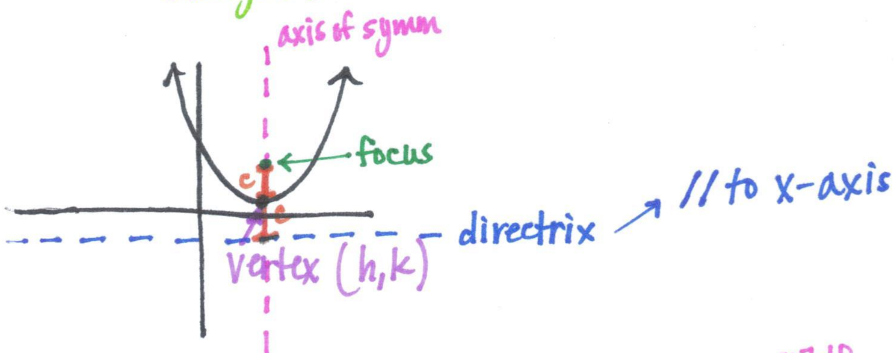
  * opens up/down
  vertex: (h,k)
  axis of symm: $x = h$

* "Focus": a point inside the parabola on the axis of symmetry.

* "Directrix": a line ⊥ to the axis of symmetry outside the parabola.

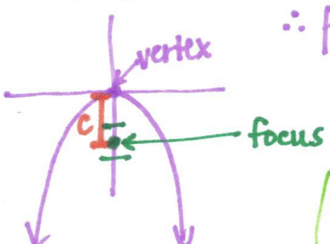
* "c": the distance from the vertex to the focus and the distance from the vertex to the directrix.

distance → $c = \left|\frac{1}{4a}\right|$ and $a = \frac{1}{4c}$
∴ always (+)



like #1-6
ex: what's the eq. for a parabola w/vertex @ (0,0) & focus @ (0,−1.5)

h k



∴ parabola must open downward

$c = 1.5 \rightarrow a = \frac{1}{4(-1.5)}$

(*use −1.5 to open downward)

$= \frac{-1}{6}$

$\boxed{\therefore\ y = -\frac{1}{6}(x-0)^2 + 0 \quad \text{or } y = -\frac{1}{6}x^2}$

like #7-10
ex: $y = \frac{x^2}{4}$ find the vertex, focus & directrix.

→ $h=0, k=0$
(0,0)

$a = \frac{1}{4}$

∴ $c = \left|\frac{1}{4(\frac{1}{4})}\right| = 1$

$c = 1$

$\boxed{\text{vertex: } (0,0)}$

$\boxed{\text{focus: } (0,1)}$

$\boxed{\text{directrix: } y = -1}$

* Parabolas can also open L/R!

$$x = a(y-k)^2 + h$$

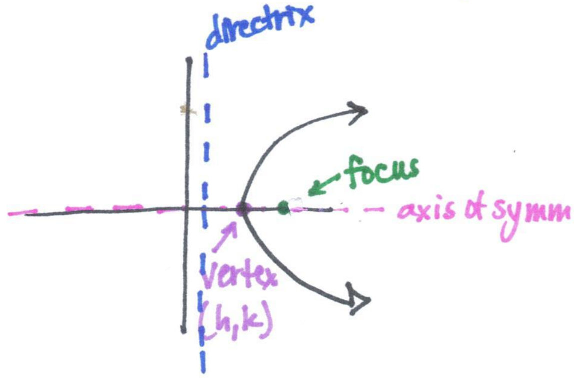
* Notice who is in the parentheses!

* opens L/R ← if "a" is pos.
(L ← if "a" is neg)

vertex: (h, k)

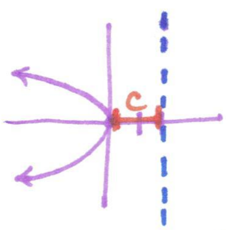
axis of symm: $y = k$

directrix is // to y-axis



like #11-16

ex: Write the eq. of parabola w/vertex @ $(0,0)$ & directrix @ $x = 2$



∴ $c = 2$ & opens L so "a" is (−).

$$a = \frac{1}{4(-2)} = \frac{-1}{8}$$

$$\boxed{x = \frac{-1}{8}(y-0)^2 + 0 \quad \text{or} \quad x = \frac{-1}{8}y^2}$$

like #18-20

ex: $y + 2 = -\frac{1}{8}(x-1)^2$ ← x is squared ∴ up/down (− ∴ downward)

Find vertex, focus, directrix & graph.

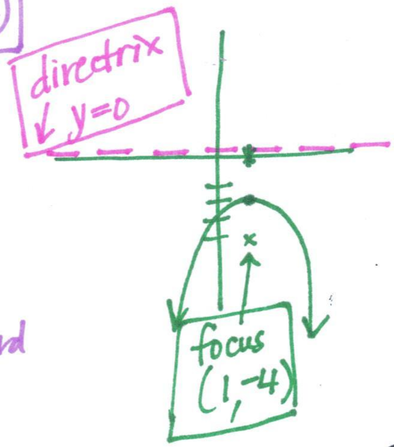
$\boxed{\text{vertex: } (1,-2)}$

$$\frac{-1}{8} = \frac{1}{4c}$$

$$-4c = 8$$

$$c = -2$$

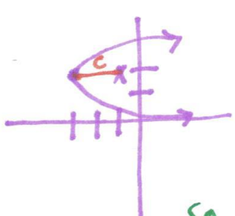
↓ opens downward



like #21-26

ex: Vertex: $(-3,2)$ focus: $(-1,2)$

Write an equation of the parabola.



∴ y is squared & a is (+)

$$c = 2 \;\therefore\; a = \frac{1}{4c} = \frac{1}{4(2)} = \frac{1}{8} = \frac{1}{8}$$

so... $\boxed{x + 3 = \frac{1}{8}(y-2)^2}$

#20 $y^2 - 4x - 2y = 3$

$$y^2 - 2y + \underline{1} = 4x + 3 + \underline{1}$$

$$\frac{-2}{2} = (-1)^2$$

$$(y-1)^2 = 4x + 4$$

$$(y-1)^2 = 4(x+1)$$

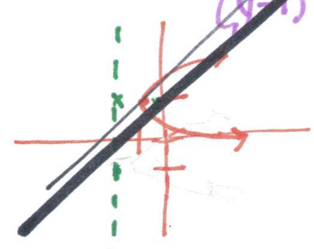
$\boxed{\text{vertex: } (-1,1)}$

opens right!

$$4 = \frac{1}{4c}$$

$$16c = 1$$

$$c = 1/16$$



$\boxed{\text{focus: } (-15/16, 1) \quad \text{directrix: } x = -17/16}$

#20 $y^2 - 4x - 2y = 3$

$y^2 - 2y + \underline{1} = 4x + 3 + \underline{1}$

$\frac{-2}{2} = (-1)^2$

$(y-1)^2 = 4x + 4$

$\frac{(y-1)^2}{4} = \frac{4(x+1)}{4}$

$\frac{1}{4}(y-1)^2 = x + 1$

$x = \frac{1}{4}(y-1)^2 - 1$

a k h

$\boxed{\text{vertex: } (-1, 1)}$

$a = \frac{1}{4}$ & y^2 ∴ opens right

$c = \left|\frac{1}{4a}\right| = \left|\frac{1}{4(1/4)}\right|$

$= \left|\frac{1}{1}\right| = 1$