Completing the Square (Section 3-6)

* This method works well when getting a quadratic equation into vertex form. → $y=(x-h)^2+k$ vertex: (h,k)

STEPS: ① Move the constant "c" to the opp. side of the equation & add blanks.
② Take half of "b" & square it.
③ Add this to both sides of the equation.
④ Factor the quadratic or use the short-cut. (It's always a perfect square trinomial)
⑤ Square root both sides. Don't forget "±"
⑥ Solve for x.

#1 on Notes

ex: $x^2+8x+12=0$

$x^2+8x+\underline{16}=-12+\underline{16}$

$\frac{8}{2}=(4)^2$ short-cut!

$\sqrt{(x+4)^2}=\sqrt{4}$

$x+4=\pm 2$
$-4 \quad -4$

$x=2-4$ or $x=-2-4$

$\boxed{x=-2}$ $\boxed{x=-6}$

#2 on Notes * No "c"!

ex: $x^2-6x=0$

$x^2-6x+\underline{9}=0+\underline{9}$

$\frac{-6}{2}=(-3)^2$ short-cut!

$\sqrt{(x-3)^2}=\sqrt{9}$

$x-3=\pm 3$
$+3 \quad +3$

$x=3+3$ or $x=-3+3$

$\boxed{x=6}$ $\boxed{x=0}$

#3 on Notes

ex: $x^2-4=0$

$\sqrt{x^2}=\sqrt{4}$ * No "b" so just sq. root!

$\boxed{x=\pm 2}$

ex: $x^2+7x+3=0$ → *Not always "Nice" numbers!

$x^2+7x+\frac{49}{4}=-3+\frac{49}{4}$

$\left(\frac{7}{2}\right)^2$

$\sqrt{(x+7/2)^2}=\sqrt{9.25}$

$x+7/2=\pm 3.0414$

$-7/2 \quad -7/2$

$\boxed{x=-6.541}$ or $\boxed{x=-.459}$

* If the quadratic has a coefficient for x^2, "a", then you must divide by "a" before you begin the process of completing the square!

#5 on Notes

ex: $\frac{2x^2+3x+1=0}{2}$

$x^2+\frac{3}{2}x+\frac{1}{2}=0$

$x^2+\frac{3}{2}x+\frac{9}{16}=-\frac{1}{2}+\frac{9}{16}$

$\frac{3/2}{2}=\left(\frac{3}{4}\right)^2$

$\sqrt{(x+3/4)^2}=\sqrt{\frac{1}{16}}$

$x+\frac{3}{4}=\pm\frac{1}{4}$

$-3/4 \quad -3/4$

$\boxed{x=-1/2}$ $\boxed{x=-1}$

#8 in WB pg 171

ex: $A=420 \quad A=l\cdot w$

$420=x(2x+5)$

$\frac{420}{2}=\frac{2x^2+5x}{2}$

$\frac{25}{16}+210=x^2+\frac{5}{2}x+\frac{25}{16}$

$\frac{5/2}{2}=\left(\frac{5}{4}\right)^2$

$\sqrt{211.5625}=\sqrt{(x+5/4)^2}$

$\pm 14.545=x+5/4$

$-5/4 \quad -5/4$

$\boxed{x=13.295}$, ~~$x=-15.795$~~

COMPLETE THE SQUARE

Another method to solve quadratic equations in the form: $ax^2+bx+c=0$

Steps:
1. The *a*-value must be 1. If it is not 1 then divide everything by *a*
2. Move the *c*-value to the opposite side of the equation.
3. Take $\frac{1}{2}$ of the *b*-value. Square that number. Add the result to both sides of the equation.
4. Factor the left side of the equation. (It should be a perfect square trinomial.)
5. Square root both sides of the equation. (Don't forget the ± sign on right side.)
6. Isolate the variable to complete the problem.

Example #1

$x^2-6x+8=0$

Factor Method: $(x-4)(x-2)=0$ → $x=4$ $x=2$

$x^2-6x=-8$

$x^2-6x+?=-8+?$

$x^2-6x+9=-8+9$

$(x-3)^2=1$

$\sqrt{(x-3)^2}=\pm\sqrt{1}$

$x-3=\pm1$

$x=3\pm1$

$x=3+1 \quad x=3-1$

$x=4 \quad x=2$

factor: it will always be a perfect square trinomial

$x^2-6x+\underline{9}=-8+\underline{9}$

$\frac{-6}{2}=(-3)^2$ short-cut

$\sqrt{(x-3)^2}=\sqrt{1}$

$x-3=\pm1$ (+3 +3)

$x=1+3$ or $x=-1+3$

$x=4$ $x=2$

Solve by completing the square:

1. $x^2+8x+12=0$

$x^2+8x+\underline{16}=-12+\underline{16}$

$\frac{+8}{2}=(+4)^2$

$\sqrt{(x+4)^2}=\sqrt{4}$

$x+4=\pm2$ (−4 −4)

$x=2-4$ or $x=-2-4$

$x=-2$ $x=-6$

$x^2+8x+12=0$

$(x+6)(x+2)=0$

$x=-6$ $x=-2$ ✓

2. $x^2-6x=0$ * No "c"

$x^2-6x+\underline{9}=0+\underline{9}$

$\frac{-6}{2}=(-3)^2$

$\sqrt{(x-3)^2}=\sqrt{9}$

$x-3=\pm3$ (+3 +3)

$x=3+3$ or $x=-3+3$

$x=6$ $x=0$

$x^2-6x=0$

$x(x-6)=0$

$x=0$ $x=6$ ✓

3. $x^2-4=0$ *No "b"

$\sqrt{x^2}=\sqrt{4}$

$x=\pm2$

4. $x^2+7x+3=0$ → on the back!

* If "a" is not 1, divide all of the equation by "a"!

Example #2

$2x^2+6x-7=0$ *Step #1*

$\frac{2}{2}x^2+\frac{6}{2}x-\frac{7}{2}=\frac{0}{2}$

$x^2+3x-\frac{7}{2}=0$ *Step #2*

$x^2+3x=\frac{7}{2}$

$x^2+3x+?=\frac{7}{2}+?$ *Step #3*

$x^2+3x+\frac{9}{4}=\frac{7}{2}+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2=\frac{7}{2}+\frac{9}{4}$ *Step #4*

$\left(x+\frac{3}{2}\right)^2=\frac{14}{4}+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2=\frac{23}{4}$

$\sqrt{\left(x+\frac{3}{2}\right)^2}=\pm\sqrt{\frac{23}{4}}$ *Step #5*

$x+\frac{3}{2}=\pm\frac{\sqrt{23}}{2}$

$x=-\frac{3}{2}\pm\frac{\sqrt{23}}{2}$ *Step #6*

#4 $x^2+7x+3=0$ → $-\frac{12}{4}$

$x^2+7x+\frac{49}{4}=-3+\frac{49}{4}$

$\left(\frac{+7}{2}\right)^2$

$\sqrt{(x+7/2)^2}=\sqrt{\frac{37}{4}}=\frac{\sqrt{37}}{\sqrt{4}}$

$x+7/2=\frac{\pm\sqrt{37}}{2}-7/2$

$-7/2$

$$x=\frac{-7\pm\sqrt{37}}{2}$$

$\frac{3/2}{2}=\frac{\frac{3}{2}}{\frac{2}{1}}=\frac{3}{2}\cdot\frac{1}{2}=\frac{3}{4}$

5. $\frac{2x^2+3x+1=0}{2}$ $\frac{0}{2}$

$x^2+\frac{3}{2}x+\frac{1}{2}=0$ → $-\frac{8}{16}$

$x^2+\frac{3}{2}x+\frac{9}{16}=\left(-\frac{1}{2}\right)+\frac{9}{16}$

$\frac{+3/2}{2}=\left(\frac{+3}{4}\right)^2$

$\sqrt{\left(x+\frac{3}{4}\right)^2}=\sqrt{\frac{1}{16}}$

$x+3/4=\pm\frac{1}{4}$

$-3/4 \quad -3/4$

$x=\frac{1}{4}-\frac{3}{4}$ or $x=\frac{-1}{4}-\frac{3}{4}$

$x=-\frac{1}{2}$ $\quad$ $x=-1$

~~6. $3x^2-5x=0$~~