Quadratic Formula & the Discriminant (Section 3-7)

** Quadratic Formula:

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ ← make sure everything is divided by 2a!

* The discriminant is "b^2-4ac", the part under the radical.

This determines the # & type of roots!

* If $b^2-4ac > 0$, then you have 2 Real Roots

If $b^2-4ac = 0$, then you have 1 Real Root

If $b^2-4ac < 0$, then you have ∅ Real Roots (you have 2 imaginary roots!)

ex: $x^2-4x=21$ Find the roots using quad. formula.

$x^2-4x-21=0$

$a=1$
$b=-4$
$c=-20$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-21)}}{2(1)}$$

b^2 is ALWAYS (+) → $16+84$ $=100$ → This is (+) ∴ 2 Real Roots!

*I always do this simplification seperate!

$$x=\frac{4\pm\sqrt{100}}{2}=\frac{4\pm10}{2}$$

↗ $\frac{4+10}{2}=\frac{14}{2}=\boxed{7}$

↘ $\frac{4-10}{2}=\frac{-6}{2}=\boxed{-3}$

ex: $3x^2-41x=-110$

$3x^2-41x+110=0$

$a=3$
$b=-41$
$c=110$

$$x=\frac{-(-41)\pm\sqrt{(-41)^2-4(3)(110)}}{2(3)}$$

b^2 is ALWAYS (+) → $1681-1320$ $=361$ → (+) ∴ 2 Real Roots!

$$x=\frac{41\pm\sqrt{361}}{6}=\frac{41\pm19}{6}$$

↗ $\frac{41+19}{6}=\frac{60}{6}=\boxed{10}$

↘ $\frac{41-19}{6}=\frac{22}{6}=\boxed{11/3}$

ex: $2x^2 - 16x = -25$

$2x^2 - 16x + 25 = 0$

$a = 2$

$b = -16$

$c = 25$

b^2 is ALWAYS (+)

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(25)}}{2(2)} \rightarrow 256 - 200 = 56 \rightarrow (+)$$

$\therefore$ 2 Real Roots!

$$x = \frac{16 \pm \sqrt{56}}{4} \nearrow \frac{16 + \sqrt{56}}{4} = \boxed{5.871}$$

$$\searrow \frac{16 - \sqrt{56}}{4} = \boxed{2.129}$$

* So when do you use each method we have learned?

- If there is no x-term → Square Roots
- If it is easily factorable → Factoring
- If there is a coefficient for x^2 → Quad Formula or Graphing
- If the equation is not factorable → Quad Formula, Completing the Square, or Graphing
- If coefficients a, b, & c are really large → Quadratic Formula or Graphing.

ex: $9x^2 + 12x + 4 = 0$ How many REAL solutions?

$a = 9$

$b = 12$

$c = 4$

$$b^2 - 4ac = (12)^2 - 4(9)(4)$$
$$= 144 - 144$$
$$= 0 \therefore \text{1 Real Solution!}$$

ex: $2x^2 - 5x + 3 = 0$ How many REAL solutions?

$a = 2$

$b = -5$

$c = 3$

$$b^2 - 4ac = (-5)^2 - 4(2)(3)$$
$$= 25 - 24$$
$$= 1 \therefore \text{2 Real Solutions!}$$