Complex Numbers (Section 3-8)

* The square root of -1 is $i \rightarrow$ an imaginary number.

ex: $\sqrt{-1} = i$ ex: $\sqrt{-16} = 4i$ ex: $\sqrt{-a} = i\sqrt{a}$

ex: $\sqrt{-25} = i\sqrt{25}$ $= 5i$

ex: $\sqrt{-81} = i\sqrt{81}$ $= 9i$

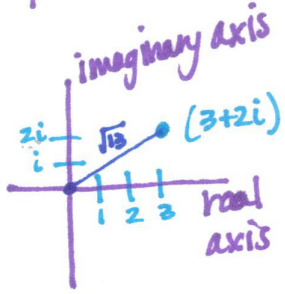
ex: $\sqrt{-50} = i\sqrt{50} < \frac{25}{2}$ $= 5i\sqrt{2}$

* Complex Numbers $(a+bi)$: have both an imaginary & a real part to them.

(a: Real part, bi: Imaginary part)

* Real Numbers: $a + 0i$
* Imagnary: $0 + bi$
* Absolute value of a complex #: $|a+bi| = \sqrt{a^2+b^2}$

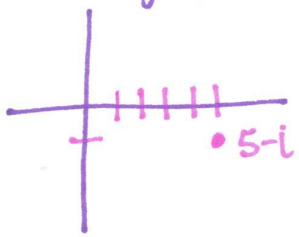
* Complex Plane:



plot $3+2i$

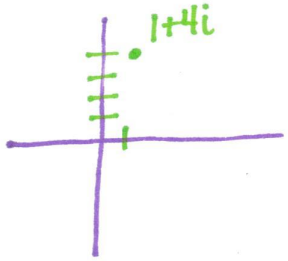
$|3+2i| = \sqrt{3^2+2^2}$
$= \sqrt{9+4}$
$= \sqrt{13}$

ex: $5-i$ graph & find absol value



$|5-i| = \sqrt{5^2+(-1)^2}$
$= \sqrt{25+1}$
$= \sqrt{26}$

ex: $1+4i$ graph & find absol. value



$|1+4i| = \sqrt{1^2+4^2}$
$= \sqrt{1+16}$
$= \sqrt{17}$

* Adding / Subtracting Complex Numbers: add/sub the real part w/ the real part & the imaginary w/ imaginary.

ex: $(7-2i) + (-3+i)$
$(7-3) + (-2i+i)$
$= 4-i$

ex: $(1+5i) + (-3+2i)$
$(1-3) + (5i+2i)$
$-2+7i$

ex: $(8+6i) + (-8+6i)$
$(8-8) + (6i+6i)$
$= 0+12i$ or $12i$

* Multiplying Complex Numbers: you must remember that $i^2 = -1$

ex: $(7i)(3i) = 21i^2$
$= 21(-1)$
$= -21$

ex: $(2-3i)(4+5i)$ FOIL
$= 8+10i-12i-15i^2$
$= 8-2i+15(+1)$
$= 23-2i$

* This is sum. diff!

ex: $(-4+5i)(-4-5i)$
$= 16+20i-20i-25i^2$
$= 16+25(+1)$
$= 41$

* Dividing Complex Numbers: you must multiply by the conjugate of the denominator.

ex: $a+bi$ Conj: $a-bi$

ex: $\frac{(5-2i)(3-4i)}{(3+4i)(3-4i)} = \frac{15-20i-6i+8i^2}{9-16i^2}$ ($i^2=-1$)

Conj = $3-4i$

$= \frac{7-26i}{25}$

ex: $\frac{(4-i)(-6i)}{(6i)(-6i)} = \frac{-24i+6i^2}{-36i^2}$ ($i^2=-1$)

conj: $-6i$

$= \frac{-6-24i}{36}$

ex: $\frac{(4-3i)(-1+4i)}{(-1-4i)(-1+4i)} = \frac{-4+16i+3i-12i^2}{1-16i^2}$ ($i^2=-1$)

Conj = $-1+4i$

$= \frac{8+19i}{17}$

ex: Factor $5x^2+20$ using conjugates.
$= 5(x^2+4)$
$= 5(x+2i)(x-2i)$

ex: x^2+81
$= (x+9i)(x-9i)$

ex: $-9x^2-100 = -1(9x^2+100)$
$= -1(3x+10i)(3x-10i)$