

# Solving Quadratic Equations

## Unit Review

Solve questions 1-6 by taking square roots:

$$1. x^2 - 81 = 0$$

$$\sqrt{x^2} = \sqrt{81}$$

$$x = \pm 9$$

$$2. 3x^2 - 192 = 0$$

$$\frac{3x^2}{3} = \frac{192}{3}$$

$$\sqrt{x^2} = \sqrt{64}$$

$$x = \pm 8$$

$$3. x^2 - 121 = 0$$

$$\sqrt{x^2} = \sqrt{121}$$

$$x = \pm 11$$

$$4. 5x^2 - 245 = 0$$

$$\frac{5x^2}{5} = \frac{245}{5}$$

$$\sqrt{x^2} = \sqrt{49}$$

$$x = \pm 7$$

$$5. x^2 + 36 = 0$$

$$\sqrt{x^2} = \sqrt{-36}$$

Imaginary!

∴ No Solution

$$6. 4x^2 - 144 = 0$$

$$\frac{4x^2}{4} = \frac{144}{4}$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = \pm 6$$

Solve questions 7-13 by factoring:

7.  $x^2 - 7x - 8 = 0$

$$(x-8)(x+1) = 0$$

$$\boxed{x=8; x=-1}$$

8.  $x^2 = -6x$

$$x^2 + 6x = 0$$

$$x(x+6) = 0$$

$$\boxed{x=0; x=-6}$$

9.  $m^2 + 8m + 7 = 0$

$$(m+7)(m+1) = 0$$

$$\boxed{m=-7; m=-1}$$

10.  $n^2 + 2n - 24 = 0$

$$(n+6)(n-4) = 0$$

$$\boxed{n=-6; n=4}$$

11.  $c^2 = 8c$

$$c^2 - 8c = 0$$

$$c(c-8) = 0$$

$$\boxed{c=0; c=8}$$

12.  $5r^2 - 11r + 2 = 0$

$$5r^2 - 10r - r + 2 = 0$$

$$(5r-1)(r-2) = 0$$

$$\boxed{r=1/5; r=2}$$

13.  $3n^2 - 13n = 30$

$$3n^2 - 13n - 30 = 0$$

$$90 < \begin{matrix} 18 \\ -5 \end{matrix}$$

$$3n^2 + 18n - 5n - 30 = 0$$

$$(3n-5)(n+6) = 0$$

$$\boxed{n=5/3; n=-6}$$

Solve questions 14-19 by completing the square:  
Round to the nearest hundredth if necessary.

$$14. r^2 + 6r = 16$$

$$r^2 + 6r + 9 = 16 + 9$$

$$\frac{6}{2} = (3)^2$$

$$\sqrt{(r+3)^2} = \sqrt{25}$$

$$r+3 = \pm 5$$

$$r = -3 \pm 5 \rightarrow \begin{matrix} -3+5=2 \\ -3-5=-8 \end{matrix}$$

$$15. m^2 - 12m + 1 = 0$$

$$m^2 - 12m + 36 = -1 + 36$$

$$\frac{-12}{2} = (-6)^2$$

$$\sqrt{(m-6)^2} = \sqrt{35}$$

$$m-6 = \pm\sqrt{35}$$

$$m = 6 \pm \sqrt{35}$$

$$16. n^2 - 5n = 3$$

$$n^2 - 5n + \frac{25}{4} = 3 + \frac{25}{4}$$

$$\left(\frac{-5}{2}\right)^2$$

$$\sqrt{\left(n - \frac{5}{2}\right)^2} = \sqrt{\frac{37}{4}}$$

$$n - \frac{5}{2} = \pm \frac{\sqrt{37}}{2}$$

$$n = \frac{5 \pm \sqrt{37}}{2}$$

$$17. v^2 + 3v - 5 = 0$$

$$v^2 + 3v + \frac{9}{4} = 5 + \frac{9}{4}$$

$$\left(\frac{3}{2}\right)^2$$

$$\sqrt{\left(v + \frac{3}{2}\right)^2} = \sqrt{\frac{29}{4}}$$

$$v + \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$$

$$v = \frac{-3 \pm \sqrt{29}}{2}$$

$$18. t^2 + 6t - 11 = 0$$

$$t^2 + 6t + 9 = 11 + 9$$

$$\frac{6}{2} = (3)^2$$

$$\sqrt{(t+3)^2} = \sqrt{20}$$

$$t+3 = \pm\sqrt{20}$$

$$t = -3 \pm \sqrt{20}$$

$$19. x^2 - 3x = -1$$

$$x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}$$

$$\left(-\frac{3}{2}\right)^2$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{5}{4}}$$

$$x - \frac{3}{2} = \pm \frac{\sqrt{5}}{2}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$



Solve questions 20-26 using quadratic formula:

Round answers to the nearest hundredth if necessary.

20.  $6g^2 + g - 2 = 0$

$a=6$   
 $b=1$   
 $c=-2$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 + 4(6)(-2)}}{2(6)} \rightarrow \frac{1 \pm \sqrt{49}}{12}$

$x = \frac{-1 \pm \sqrt{49}}{12} \rightarrow \frac{-1+7}{12} = \frac{6}{12} = \frac{1}{2}$   
 $\frac{-1-7}{12} = \frac{-8}{12} = \frac{-2}{3}$

21.  $a^2 + 7a - 9 = 0$

$a=1$   
 $b=7$   
 $c=-9$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 + 4(1)(-9)}}{2(1)} \rightarrow \frac{7 \pm \sqrt{49-36}}{2}$

$x = \frac{-7 \pm \sqrt{85}}{2}$

22.  $x^2 - 4x - 7 = 0$

$a=1$   
 $b=-4$   
 $c=-7$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 + 4(1)(-7)}}{2(1)} \rightarrow \frac{4 \pm \sqrt{16-28}}{2}$

$x = \frac{4 \pm \sqrt{44}}{2}$

23.  $2x^2 - 5x = 12 \rightarrow 2x^2 - 5x - 12 = 0$

$a=2$   
 $b=-5$   
 $c=-12$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 + 4(2)(-12)}}{2(2)} \rightarrow \frac{5 \pm \sqrt{25-96}}{4}$

$x = \frac{5 \pm \sqrt{121}}{4} \rightarrow \frac{5+11}{4} = \frac{16}{4} = 4$   
 $\frac{5-11}{4} = \frac{-6}{4} = \frac{-3}{2}$

24.  $p^2 - 11p = -1 \rightarrow p^2 - 11p + 1 = 0$

$a=1$   
 $b=-11$   
 $c=1$

$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(1)}}{2(1)} \rightarrow \frac{11 \pm \sqrt{121-4}}{2}$

$x = \frac{11 \pm \sqrt{117}}{2}$

25.  $2x^2 + 10x = -3 \rightarrow 2x^2 + 10x + 3 = 0$

$a=2$   
 $b=10$   
 $c=3$

$x = \frac{-10 \pm \sqrt{10^2 - 4(2)(3)}}{2(2)} \rightarrow \frac{-10 \pm \sqrt{100-24}}{4}$

$x = \frac{-10 \pm \sqrt{76}}{4}$

26. A water balloon is tossed into the air with an upward velocity of 25 ft/s. Its height  $h(t)$  in ft after  $t$  seconds is given by the function  $h(t) = -16t^2 + 25t + 3$ .

a. After how many seconds will the balloon hit the ground?

b. What will the height be at  $t = 1$  second?

$h = 12$

$\hookrightarrow y_2 = 0$  intersect  $y_1$   
 $t = 1.674 \text{ sec}$

$x_{\min} = -5$   $y_{\min} = -5$   
 $x_{\max} = 5$   $y_{\max} = 15$

For questions 27-30, calculate the discriminant and then identify the number and type of solutions:

27.  $2h^2 - 3h + 2 = 0$

$a=2$   
 $b=-3$   
 $c=2$

$b^2 - 4ac = (-3)^2 - 4(2)(2) = 9 - 16 = -7$

$\therefore$  2 imaginary roots

28.  $b^2 + 5b - 3 = 0$

$a=1$   
 $b=5$   
 $c=-3$

$b^2 - 4ac = (5)^2 - 4(1)(-3) = 25 + 12 = 37$

$\therefore$  2 Real Roots

29.  $5x^2 + x + 6 = 0$

$a=5$   
 $b=1$   
 $c=6$

$b^2 - 4ac = (1)^2 - 4(5)(6) = 1 - 120 = -119$

$\therefore$  2 imag. roots

30.  $-3x^2 - 4x + 1 = 0$

$a=-3$   
 $b=-4$   
 $c=1$

$b^2 - 4ac = (-4)^2 - 4(-3)(1) = 16 + 12 = 28$

$\therefore$  2 Real Roots

Solve each system of equations in questions 31-38 by graphing, substitution, or elimination: (at least 2 for each type!)

Elim

$$31. \begin{cases} y = x^2 - 2x - 1 \\ (y = x - 1) \cdot -1 \end{cases}$$

$$(0, -1) \neq (3, 2)$$

$$\begin{array}{r} y = x^2 - 2x - 1 \\ -y = -x + 1 \\ \hline 0 = x^2 - 3x \\ x(x-3) \quad x=0, 3 \end{array}$$

$$\begin{aligned} y &= 0 - 1 = -1 \\ y &= 3 - 1 = 2 \end{aligned}$$

$$32. \begin{cases} y_1 = -3x^2 + x + 4 \\ y_2 = -2x - 2 \end{cases}$$

Graph!

Zoom #6

2nd Trace #5

scroll Enter x3

$$(-1, 0) \neq (2, -6)$$

Sub

$$33. \begin{cases} y = x^2 + 4x - 6 \\ y = x - 2 \end{cases}$$

$$(-4, -6) \neq (1, -1)$$

$$\begin{array}{r} x - 2 = x^2 + 4x - 6 \\ -x + 2 \quad \quad -x + 2 \\ \hline 0 = x^2 + 3x - 4 \\ (x+4)(x-1) \quad x = -4, 1 \end{array}$$

$$\begin{aligned} y &= 4 - 2 = -6 \\ y &= 1 - 2 = -1 \end{aligned}$$

Graph!

$$34. \begin{cases} y_1 = x^2 - x + 2 \\ y_2 = x + 5 \end{cases}$$

$$(-1, 4) \neq (3, 8)$$

Sub

$$35. \begin{cases} y = x - 3 \\ y = x^2 - 5x + 5 \end{cases}$$

$$(4, 1) \neq (2, -1)$$

$$\begin{array}{r} x - 3 = x^2 - 5x + 5 \\ -x + 3 \quad \quad -x + 3 \\ \hline 0 = x^2 - 6x + 8 \\ 0 = (x-4)(x-2) \\ x = 4, 2 \end{array}$$

$$\begin{aligned} y &= 4 - 3 = 1 \\ y &= 2 - 3 = -1 \end{aligned}$$

Elim

$$36. \begin{cases} y = 2x + 2 \\ (y = -x^2 + 6x + 7) \cdot -1 \end{cases}$$

$$(5, 12) \neq (-1, 0)$$

$$\begin{array}{r} y = 2x + 2 \\ -y = x^2 - 6x - 7 \\ \hline 0 = x^2 - 4x - 5 \\ 0 = (x-5)(x+1) \\ x = 5, -1 \end{array}$$

$$\begin{aligned} y &= 2(5) + 2 = 12 \\ y &= 2(-1) + 2 = 0 \end{aligned}$$

Elim

$$37. \begin{cases} (y = -x^2 + 5x + 1) \cdot -1 \\ y = 2x - 3 \end{cases}$$

$$(4, 5) \neq (-1, -5)$$

$$\begin{array}{r} -y = x^2 - 5x - 1 \\ y = 2x - 3 \\ \hline 0 = x^2 - 3x - 4 \\ 0 = (x-4)(x+1) \quad x = 4, -1 \end{array}$$

$$\begin{aligned} y &= 2(4) - 3 = 5 \\ y &= 2(-1) - 3 = -5 \end{aligned}$$

Sub/Graph

$$38. \begin{cases} y = x^2 - x + 2 \\ y = 2x^2 + x + 6 \end{cases}$$

$$\begin{array}{r} 2x^2 + x + 6 = x^2 - x + 2 \\ -x^2 + x - 2 \quad -x^2 + x - 2 \\ \hline x^2 + 2x + 4 = 0 \end{array}$$

$$x^2 + 2x + 4 = 0$$

Not Factorable!

No Solution

\*The graphs never intersect



Solve questions 39-49 using any method:  
Round to the nearest hundredth if necessary.

**F** 39.  $d^2 - d = 30$

$$d^2 - d - 30 = 0$$

$$(d-6)(d+5)$$

$$d = 6, -5$$

**F** 41.  $x^2 + 6x + 5 = 0$

$$(x+5)(x+1) = 0$$

$$x = -5, -1$$

**F** 43.  $f^2 + 12f = 0$

$$f(f+12) = 0$$

$$f = 0, -12$$

**F** 45.  $x^2 = 10x$

$$x^2 - 10x = 0$$

$$x(x-10) = 0$$

$$x = 0, 10$$

47.  $6x^2 - 8x - 30 = 0$

Graph!  $y_1 = 6x^2 - 8x - 30$ ;  $y_2 = 0$

$$x = -1.6, 3 \text{ or } -\frac{5}{3}, 3$$

**QF** 49.  $2x^2 + 5x - 63 = 0$

$$\begin{aligned} a &= 2 \\ b &= 5 \\ c &= -63 \end{aligned}$$

$$x = \frac{-5 \pm \sqrt{5^2 + 4(2)(63)}}{2(2)}$$

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{529}}{4} \\ &\rightarrow \frac{-5 + 23}{4} = \frac{18}{4} = 9/2 \\ &\rightarrow \frac{-5 - 23}{4} = \frac{-28}{4} = -7 \end{aligned}$$

40.  $15x^2 - 23x + 4 = 0$

Graph!  $y_1 = 15x^2 - 23x + 4$   
 $y_2 = 0$

$$x = .2, 1.3 \text{ or } \frac{1}{5}, \frac{4}{3}$$

**QF** 42.  $3x^2 - 12x = -1 \rightarrow 3x^2 - 12x + 1 = 0$

$$\begin{aligned} a &= 3 \\ b &= -12 \\ c &= 1 \end{aligned}$$

$$x = \frac{+(-12) \pm \sqrt{(-12)^2 - 4(3)(1)}}{2(3)} \rightarrow \frac{12 \pm 12}{6} = 132$$

$$x = \frac{12 \pm \sqrt{132}}{6}$$

**Solve?**

44.  $(x-5)(2x+1) = 0$

$$x = 5, -\frac{1}{2}$$

**F** 46.  $x^2 - 7x = -12$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$$x = 4, 3$$

**SqR**

48.  $x^2 - 196 = 0$

$$\sqrt{x^2} = \sqrt{196}$$

$$x = \pm 14$$

50.

The area of a rectangular soccer field is 5000 yd<sup>2</sup>. The length of the field is twice the width. Find the dimensions of the field.

$$A = 5,000$$

$$l = 2w$$

$$A = l \cdot w$$

$$\frac{5000}{2} = (2w)(w) = \frac{2w^2}{2}$$

$$\sqrt{2500} = \sqrt{w^2}$$

$$w = 50$$

$$l = 100$$