H-2

Ellipses

What you'll learn about

- Geometry of an Ellipse
- Translations of Ellipses
- Orbits and Eccentricity
- Reflective Property of an Ellipse

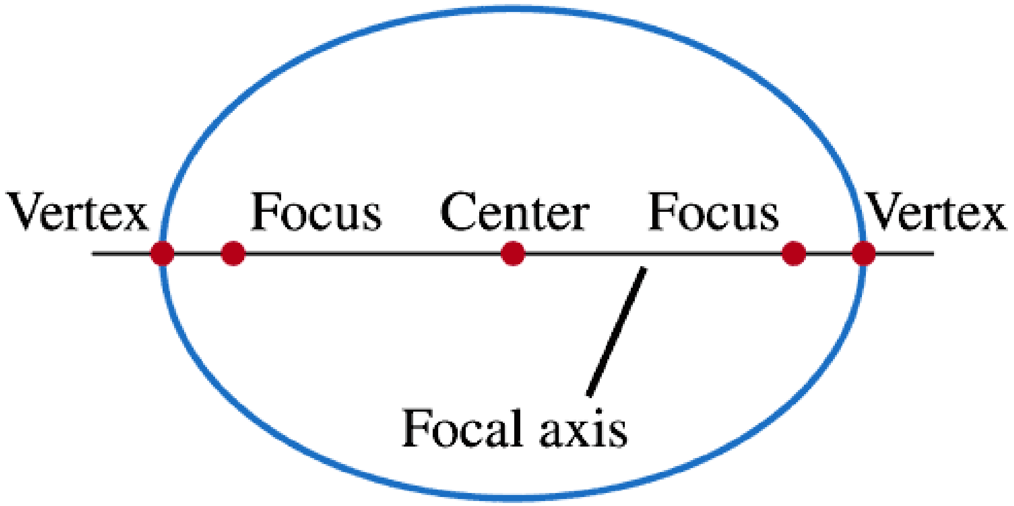
… and why

Ellipses are the paths of planets and comets around the Sun, or of moons around planets.

Ellipse

An **ellipse** is the set of all points in a plane whose distance from two fixed points in the plane have a constant sum. The fixed points are the **foci** (plural of focus) of the ellipse. The line through the foci is the **focal axis**. The point on the focal axis midway between the foci is the **center**. The points where the ellipse intersects its axis are the **vertices** of the ellipse.

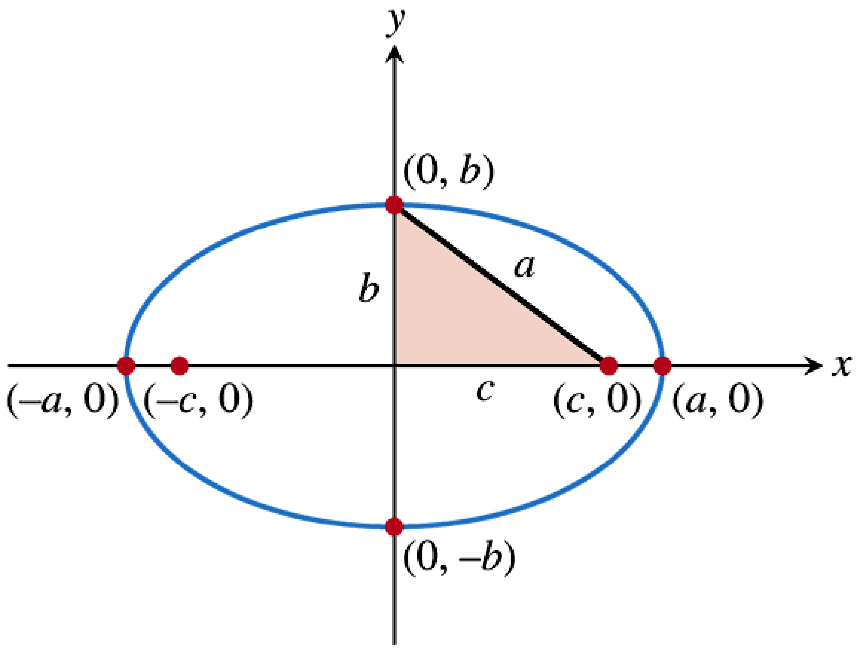
Key Points on the Focal Axis of an Ellipse



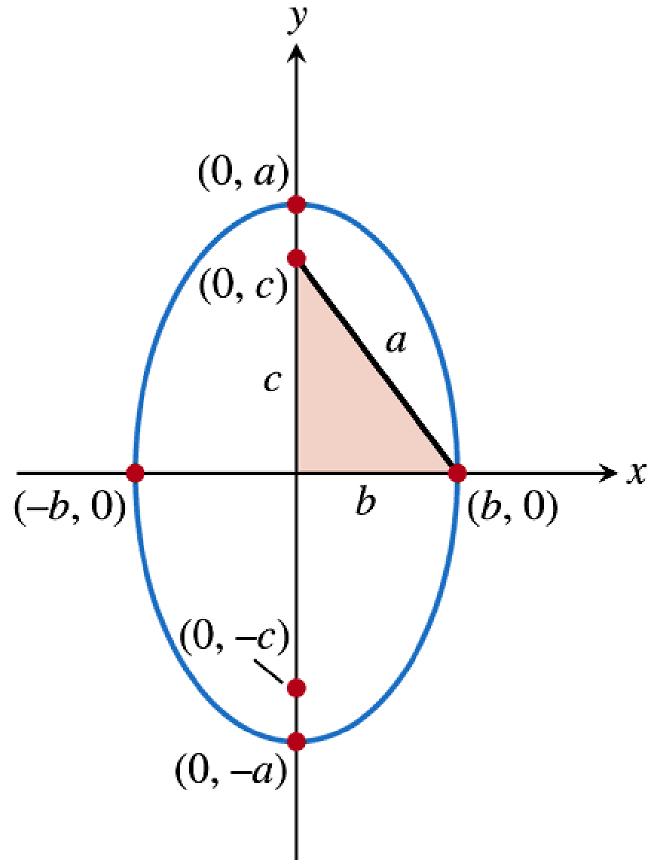
Ellipse with Center (0,0)

• Standard equation	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$	$\dfrac{y^2}{a^2}+\dfrac{x^2}{b^2}=1$
• Focal axis	x-axis	y-axis
• Foci	$(\pm c, 0)$	$(0, \pm c)$
• Vertices	$(\pm a, 0)$	$(0, \pm a)$
• Semimajor axis	a	a
• Semiminor axis	b	b
• Pythagorean relation	$a^2 = b^2 + c^2$	$a^2 = b^2 + c^2$

Pythagorean Relation



(a)



(b)

Example **Finding the Vertices and Foci of an Ellipse**

Find the vertices and the foci of the ellipse $9x^2 + 4y^2 = 36.$

Example Finding the Vertices and Foci of an Ellipse

Find the vertices and the foci of the ellipse $9x^2 + 4y^2 = 36$.

Divide both sides by 36 to put the equation in standard form.

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Since the larger number is the denominator of the y^2,

the focal axis is the y-axis. So $a^2 = 9,\ b^2 = 4,$ and

$c^2 = a^2 - b^2 = 5.$

Thus the vertices are $(0, \pm 3)$, and the foci are $(0, \pm\sqrt{5})$.

Example **Finding an Equation of an Ellipse**

Find an equation of the ellipse with foci $(-2,0)$ and $(2,0)$ whose minor axis has length 2.

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Find an equation of the ellipse with foci $(-2,0)$ and $(2,0)$ whose minor axis has length 2.

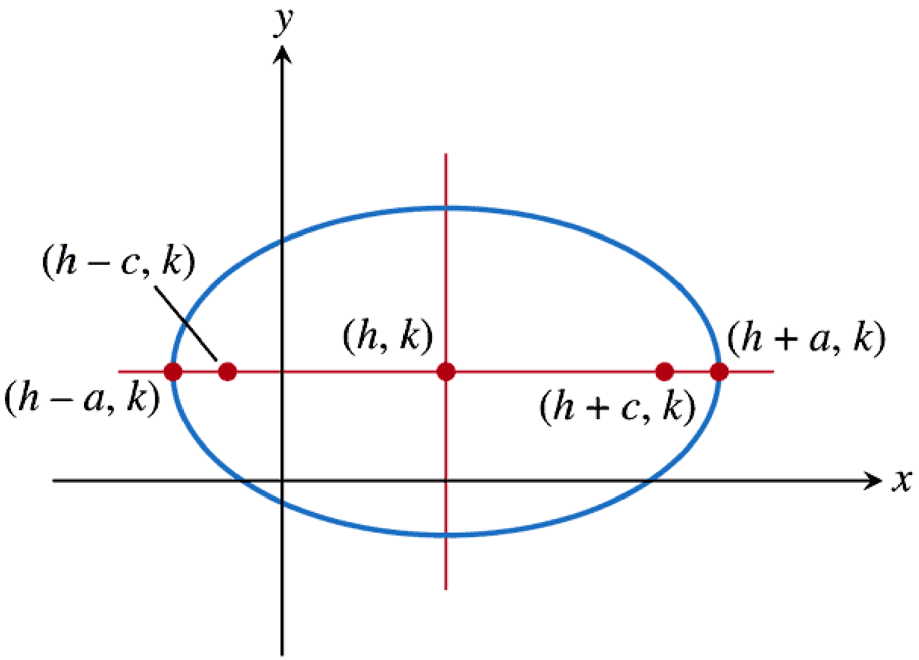
The center is $(0,0)$. The foci are on the x-axis with $c = 2$. The semiminor axis is $b = 2/2 = 1$. Using $a^2 = b^2 + c^2$, find $a^2 = 1^2 + 2^2 = 5$. Thus the equation of the ellipse is

$$\frac{x^2}{5} + \frac{y^2}{1} = 1.$$

Ellipse with Center (h,k)

Standard equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Focal axis	$y=k$	$x=h$
Foci	$(h\pm c,k)$	$(h,k\pm c)$
Vertices	$(h\pm a,k)$	$(h,k\pm a)$
Semimajor axis	a	a
Semiminor axis	b	b
Pythagorean relation	$a^2=b^2+c^2$	$a^2=b^2+c^2$

Ellipse with Center (*h*,*k*)



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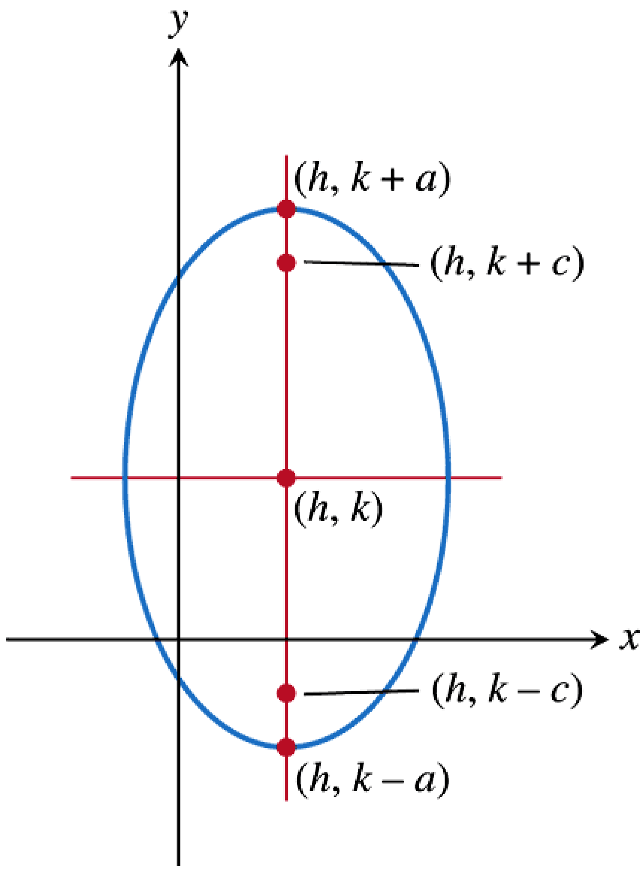

(b)

Example **Finding an Equation of an Ellipse**

Find the standard form of the equation for the ellipse whose minor axis has endpoints (-1, 4) and (5, 4), and whose major axis has length 8.

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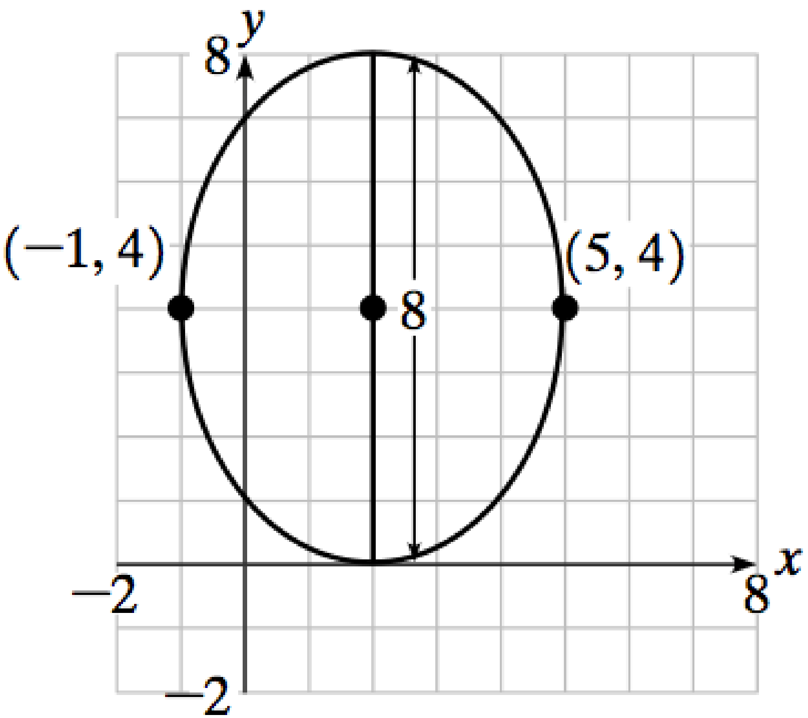
The figure shows the ellipse.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Center is midpoint of minor axis, (2, 4).



Example Finding an Equation of an Ellipse

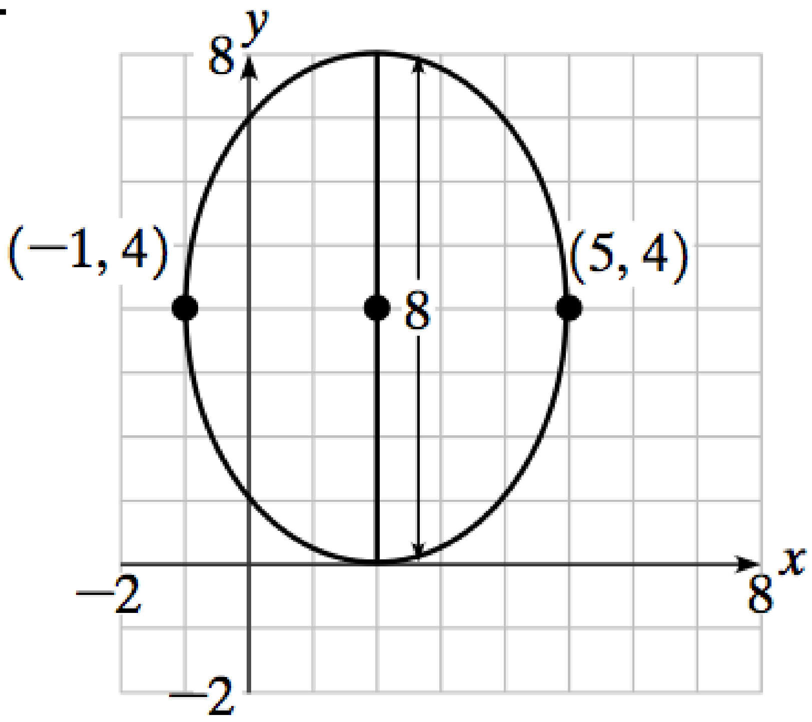
The seminor axis and semimajor axis are

$$a = \frac{5-(-1)}{2} = 3 \text{ and } b = \frac{8}{2} = 4$$

The equation we seek is

$$\frac{(x-2)^2}{3^2} + \frac{(y-4)^2}{4^2} = 1$$

$$\frac{(x-2)^2}{9} + \frac{(y-4)^2}{16} = 1$$



Example Locating Key Points of an Ellipse

Find the center, vertices, and foci of the ellipse

$$\frac{(x+1)^2}{4} + \frac{(y-1)^2}{9} = 1$$

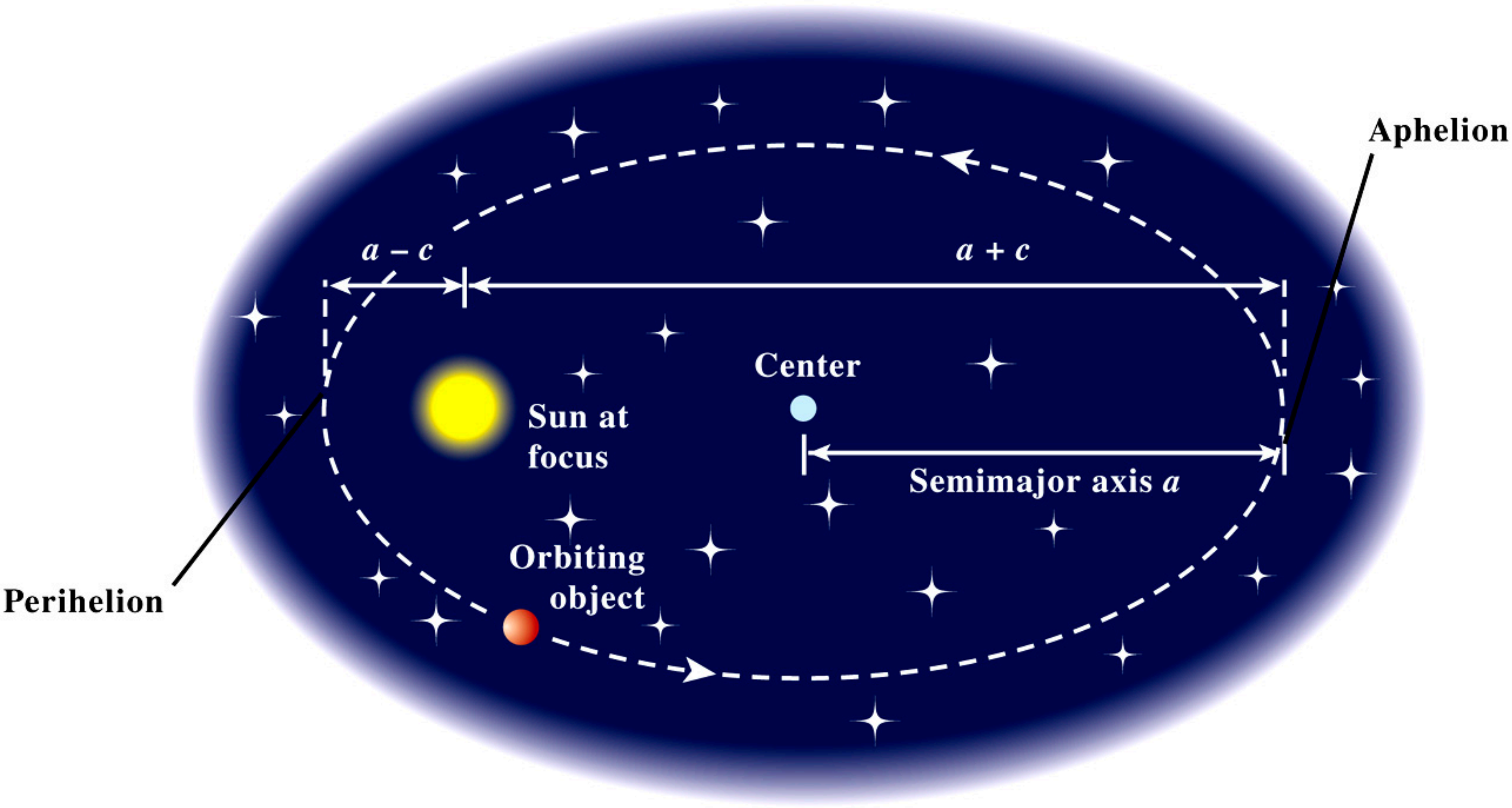
Example **Locating Key Points of an Ellipse**

The standard form of the equations is

$$\frac{(y-1)^2}{9}+\frac{(x+1)^2}{4}=1.$$

The center is at $(-1,1)$. Because the semimajor axis $a=\sqrt{9}=3,$ the vertices are at $(h,k\pm a)=(-1,1\pm 3)$ which are $(-1,4)$ and $(-1,-2)$. Because $c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5},$ the foci $(h,k\pm c)$ are $(-1,1\pm\sqrt{5})$ or approximately $(-1,3.24)$ and $(-1,-1.24)$.

Elliptical Orbits Around the Sun



Eccentricity of an Ellipse

The **eccentricity** of an ellipse is $e = \dfrac{c}{a} = \dfrac{\sqrt{a^2 - b^2}}{a}$,

where a is the semimajor axis, b is the semiminor axis, and c is the distance from the center of the ellipse to either focus.

Quick Review

1. Find the distance between (a, b) and $(1,2)$.

2. Solve for y in terms of x. $\frac{y^2}{9} + \frac{x^2}{4} = 1$

Solve for x algebraically.

3. $\sqrt{3x-8} + \sqrt{3x+12} = 10$

4. $\sqrt{6x^2+1} + \sqrt{6x^2+12} = 11$

5. Find the exact solution by completing the square.

$2x^2 + 8x - 21 = 0$

Quick Review Solutions

1. Find the distance between (a,b) and $(1,2)$. $\sqrt{(1-a)^2+(2-b)^2}$

2. Solve for y in terms of x. $\frac{y^2}{9}+\frac{x^2}{4}=1$ $y=\pm\frac{\sqrt{36-9x^2}}{2}$

Solve for x algebraically.

3. $\sqrt{3x-8}+\sqrt{3x+12}=10$ $x=8$

4. $\sqrt{6x^2+1}+\sqrt{6x^2+12}=11$ $x=\pm 2$

5. Find the exact solution by completing the square.

$2x^2+8x-21=0$ $x=-2\pm\sqrt{\frac{29}{2}}$