Logs

* Logs: $\log x = \log_{10} x$ "log base 10 of x"; we usually don't write the "10"

ex: $\log_8 x$, $\log_e x = \ln x$ (special one), $\log_b x$

* Log Properties (Just a few):

① $\log x^n = n \cdot \log x$

② $\log(x \cdot y) = \log x + \log y$

③ $\log\left(\frac{x}{y}\right) = \log x - \log y$

condensed form ← → expanded form

ex: $\log \sqrt{x} = \log x^{1/2} = \frac{1}{2} \cdot \log x$

ex: $\frac{1}{2}\log_7 y - 3\log_7 x + \log_7 z$

$= \log_7 y^{1/2} - \log_7 x^3 + \log_7 z$

$= \log_7\left(\frac{\sqrt{y} \cdot z}{x^3}\right)$

* Put exponents back up then anything added goes in the numerator and anything subtracted in the denominator.

ex: $\log_3(3^2 \cdot y^3)$

* Expand first then move the exponents to the front.

$= \log_3 3^2 + \log_3 y^3$

$= 2 \cdot \log_3 3 + 3 \cdot \log_3 y$

ex: $\ln\left(\sqrt{x^5} \cdot z^4\right)$

$= \ln x^{5/2} + \ln z^4$

$= \frac{5}{2}\ln x + 4 \ln z$

ex: $\frac{4}{3}\ln x - \frac{1}{3}\ln y$

$= \ln x^{4/3} - \ln y^{1/3}$

$= \ln\left(\frac{\sqrt[3]{x^4}}{\sqrt[3]{y}}\right) = \ln\left(\sqrt[3]{\frac{x^4}{y}}\right)$

ex: $\frac{1}{5}(\log x + \log y - 2\log z)$

$= \log x^{1/5} + \log y^{1/5} - \log z^{2/5}$

$= \log\left(\sqrt[5]{\frac{x \cdot y}{z^2}}\right)$

ex: $\ln\left(\frac{\sqrt{5}}{7^3}\right)$

$= \ln 5^{1/2} - \ln 7^3$

$= \frac{1}{2}\ln 5 - 3 \ln 7$