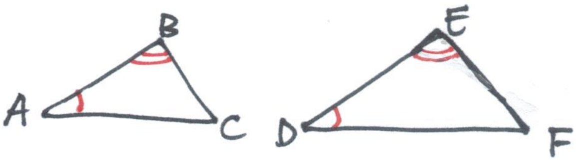
Proving Triangles Similar (Section 10-3)

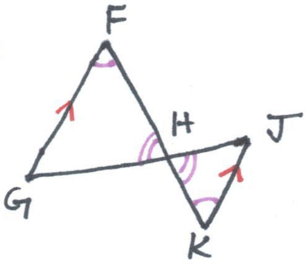
* AA~ Postulate: If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then

Angle-Angle Similarity

$\triangle ABC \sim \triangle DEF$ ← Writing the triangle in the correct order is important! (Similar angles must be in the same order.)



#1 ex:



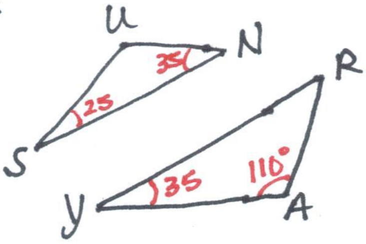
Are these similar?

$\angle F \cong \angle K$ Alt. Int. Angles

$\angle FHG \cong \angle KHJ$ Vert. Angles

$\therefore$ by AA~ $\triangle FHG \sim \triangle KHJ$

#2 ex



Are these similar?

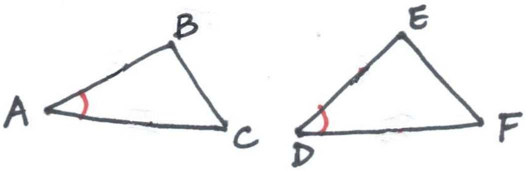
$\angle U = 180 - (25 + 35)$

$= 120°$

$\therefore$ No these are not similar.

* SAS Similarity: If $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A \cong \angle D$, then $\triangle ABC \sim \triangle DEF$

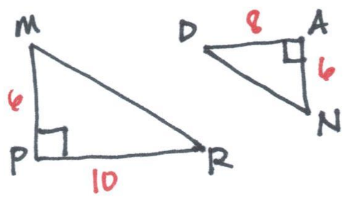
Side-Angle-Side



* SSS Similarity: If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then $\triangle ABC \sim \triangle DEF$

Side-Side-Side

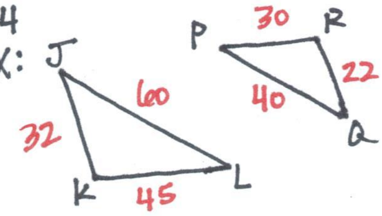
#3 ex:



$\frac{10}{8} \neq \frac{6}{6}$

$\therefore$ Not similar

#4 ex:



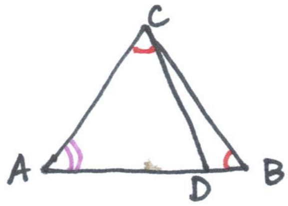
$\therefore$ Not similar

$\frac{60}{40} = \frac{45}{30} = \frac{32}{22} \rightarrow \frac{3}{2} = \frac{3}{2} \neq \frac{16}{11}$

#5

ex: Given: $\angle ABC \cong \angle ACD$

Prove: $\triangle ABC \sim \triangle ACD$



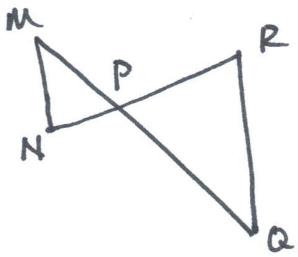
Statement	Reason
$\angle ABC \cong \angle ACD$	Given
$\angle A \cong \angle A$	Reflexive
$\triangle ABC \sim \triangle ACD$	AA~

#6

ex: Given: $PR = 2NP$

$PQ = 2MP$

Prove: $\triangle MNP \sim \triangle QRP$

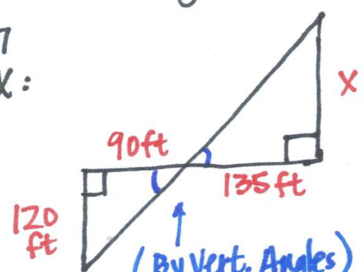


Statement	Reason
$PR = 2NP$, $PQ = 2MP$	Given
$\angle MPN \cong \angle QPR$	Vertical Angles
$\triangle MNP \sim \triangle QRP$	SAS ~

* You can use the proportions in similar triangles to find lengths of measure.

#7

ex:



(By Vert. Angles)

so the triangles are similar by AA~

So... $\dfrac{90}{135} = \dfrac{120}{x}$

$\dfrac{120 \cdot 135}{90} = \dfrac{90x}{90}$

$x = 180$ ft.

#8

ex: 1.8 m person casts 0.7 m shadow. Washington Monument casts 65.8 m shadow. How tall is the monument?

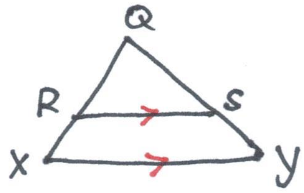
$\dfrac{1.8}{.7} = \dfrac{x}{65.8}$

$\dfrac{(1.8)(65.8)}{.7} = \dfrac{.7x}{.7}$

$\boxed{x = 169.2\text{m}}$

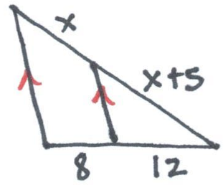
Proportions in Triangles (Section 10-5)

* Side-Splitter Theorem: If a line is // to one side of a triangle & intersects the other two sides, then it divides those sides proportionally



If $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$, then $\frac{XR}{RQ} = \frac{YS}{SQ}$

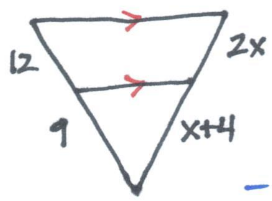
#1
ex:



$$\frac{8}{12} = \frac{x}{x+5}$$

$$8x + 40 = 12x$$
$$-8x \quad\quad -8x$$
$$\frac{40}{4} = \frac{4x}{4}$$
$$\boxed{x = 10}$$

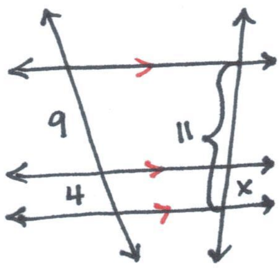
#2
ex:



$$\frac{12}{9} = \frac{2x}{x+4}$$

$$12x + 48 = 18x$$
$$-12x \quad\quad -12x$$
$$\frac{48}{6} = \frac{6x}{6}$$
$$\boxed{x = 8}$$

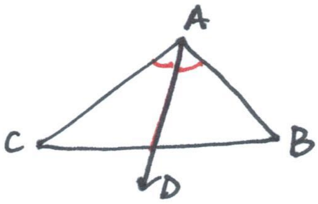
ex:



$$\frac{9}{4} = \frac{11-x}{x}$$

$$9x = 44 - 4x$$
$$+4x \quad\quad +4x$$
$$\frac{13x}{13} = \frac{44}{13}$$
$$\boxed{x = 3.385}$$

* Triangle-Angle-Bisector Theorem: If a ray bisects the angle of a triangle, then it divides the opp. side into two proportional segments to the other sides of the Δ.

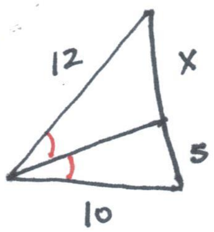


If $\overrightarrow{AD}$ bisects $\angle CAB$, then

$$\frac{CD}{DB} = \frac{CA}{BA}$$

#5

ex:



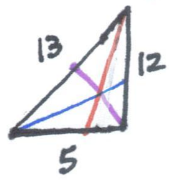
$$\frac{x}{5} = \frac{12}{10}$$

$$\frac{10x}{10} = \frac{60}{10}$$

$$\boxed{x=6}$$

#6

ex: lengths: 5, 12, 13 cm



5 cm side: $\frac{13}{12} = \frac{x}{5-x}$

$65-13x = 12x$ $\quad 65 = 25x$

$x = 2.6, 2.4$ cm

12 cm side: $\frac{13}{5} = \frac{x}{12-x}$

$156-13x = 5x$

$156 = 18x$

$x = 8.\overline{6}, 3.\overline{3}$ cm

13 cm side: $\frac{5}{12} = \frac{x}{13-x}$

$65-5x = 12x$

$65 = 17x$

$x = 3.8, 9.2$ cm