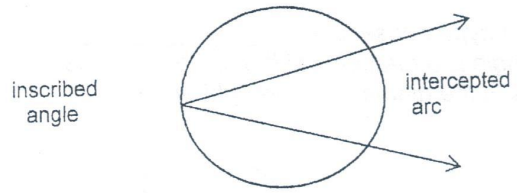
Secondary Math II - NOTES 12-5 Date _____

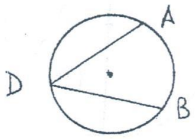
Inscribed Angles, Polygons, and Measures of Angles

An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

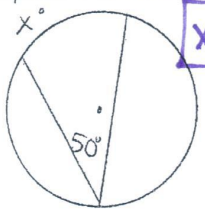
The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the intercepted arc of the angle.



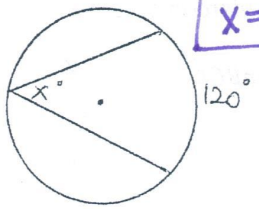
Rule: *If an angle is inscribed in a circle, then its measure is* half *the measure of its intercepted arc.* $m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$



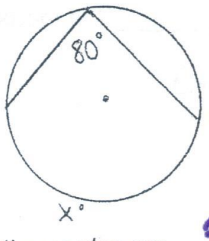
Examples: Find the measure of the arc or angle.



X=100°

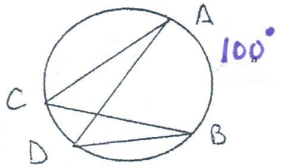


X=60°



X=160°

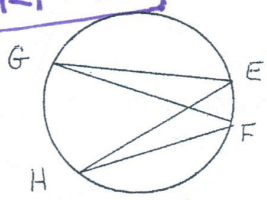
Rule: *If two inscribed angles of a circle intercept the same arc, then the angles are* ≅ (congruent).



∠C = 50°
∠D = 50°

Example: It is given that $m\angle E = 75°$. What is $m\angle F$?

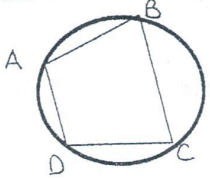
m∠F = 75°



If you were given GH = 120°, then what are the measures of E and F?

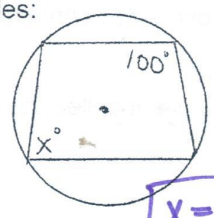
m∠E = 60°; m∠F = 60°

If all of the vertices of a polygon lie on a circle, the polygon is inscribed in the circle and the circle is circumscribed about the polygon.

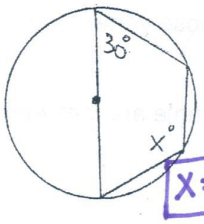


Rule: *If a quadrilateral is inscribed in a circle then the opposite angles are* supplementary.

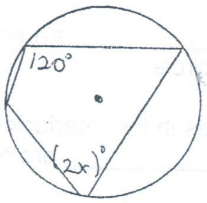
Examples:



$x = 80°$



$x = 150°$

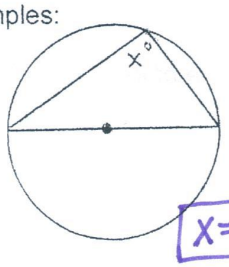


$2x = 60°$

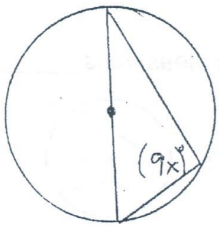
$x = 30°$

Rule: *If a right triangle is inscribed in a circle, then the hypotenuse is a* diameter *of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a* right *triangle and the angle opposite the diameter is the right angle.*

Examples:

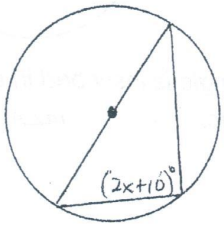


$x = 90°$



$9x = 90$

$x = 10°$



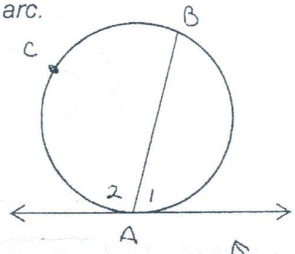
$2x + 10 = 90°$

$2x = 80°$

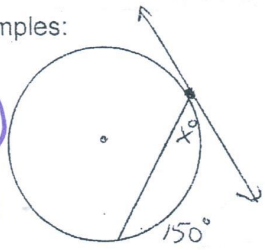
$x = 40°$

Rule: *If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one* half *the measure of its intercepted arc.*

$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $\quad$ $m\angle 2 = \frac{1}{2} m\overset{\frown}{BCA}$

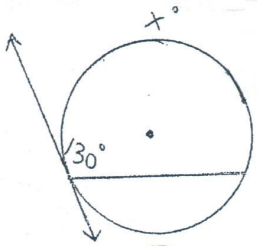


Examples:



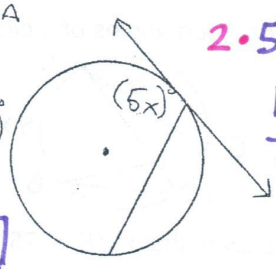
$x = \frac{1}{2}(150)$

$x = 75°$



$130 = \frac{1}{2} x$

$x = 260°$



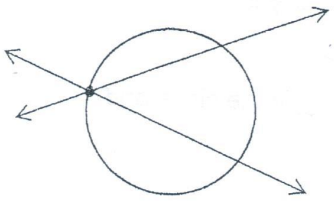
$2 \cdot 5x = \left[\frac{1}{2}(9x+20)\right] \cdot 2$

$10x = 9x + 20$

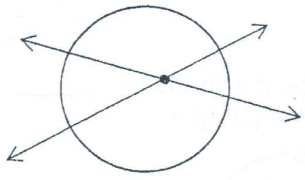
$-9x \quad -9x$

$x = 20°$

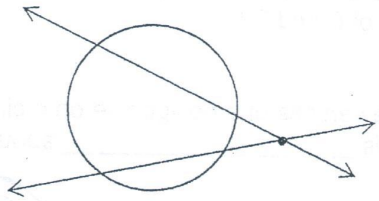
If two lines intersect a circle, there are 3 places where the lines can intersect:



On the circle



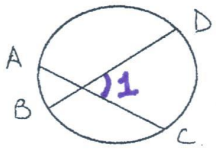
Inside the circle



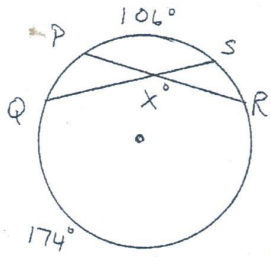
Outside the circle

Rule: *If two chords intersect in the interior of a circle, then the measure of each angle is one half the* **sum** *of the measures of the arcs intercepted by the angle and its vertical angle.*

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$



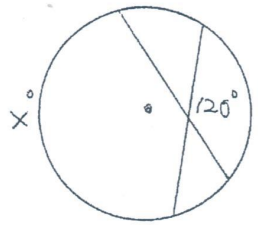
Examples:



$X = \frac{1}{2}(106 + 174)$

$X = \frac{1}{2}(280)$

$\boxed{X = 140°}$



$2 \cdot 120° = \frac{1}{2}(X + 100) \cdot 2$

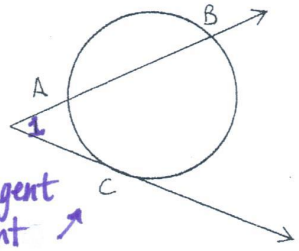
$240° = X + 100$

$100° \quad -100 \quad -100$

$\boxed{X = 140°}$

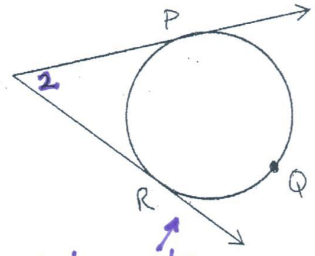
A **secant** line is a line that intersects a circle in two points. It contains a chord.

Rule: *If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is one half the* **difference** *of the measures of the intercepted arcs.*



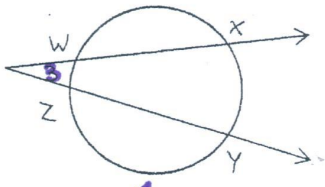
1 tangent
1 secant

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



2 tangents

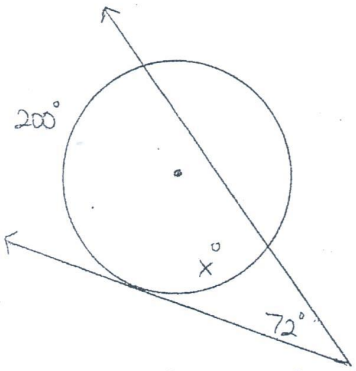
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



2 secants

$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

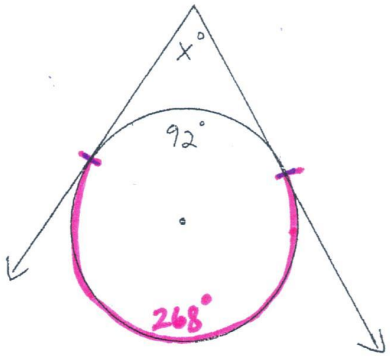
Examples:



$2 \cdot 72° = \frac{1}{2}(200 - X) \cdot 2$

$144 = 200 - X$

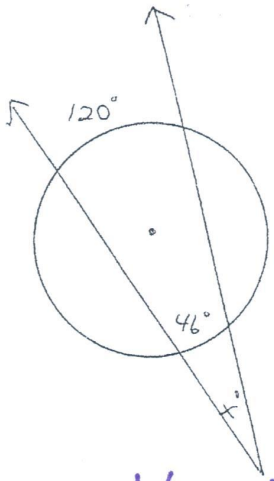
$\boxed{X = 56°}$



$X = \frac{1}{2}(268 - 92)$

$X = \frac{1}{2}(176)$

$\boxed{X = 88°}$



$X = \frac{1}{2}(120 - 46)$

$X = \frac{1}{2}(74)$

$\boxed{X = 37°}$