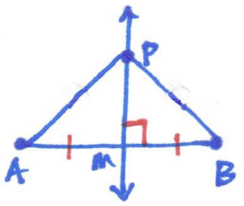
Perpendicular & Angle Bisectors (Section 8-2)

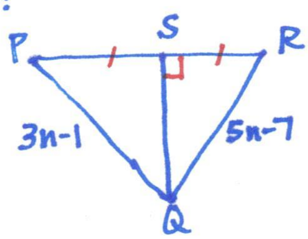
* **Perpendicular Bisector Theorem:** If a point is on a $\perp$ bisector of a segment, then it is equidistant from the end-pts.



ex: If $\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$, then $PA = PB$

* **Converse of $\perp$ Bisector Theorem:** If $PA = PB$, then $\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$

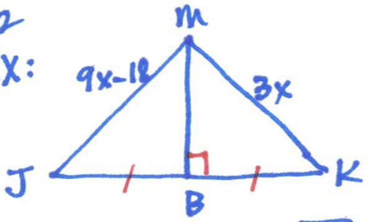
Got it?
ex:



Find length of $\overline{QR}$.

$$3n-1 = 5n-7$$
$$-3n+7 \quad -3n+7$$
$$\frac{6}{2} = \frac{2n}{2}$$
$$n = 3 \rightarrow \overline{QR} = 5(3)-7$$
$$\boxed{\overline{QR} = 8}$$

\#2
ex:

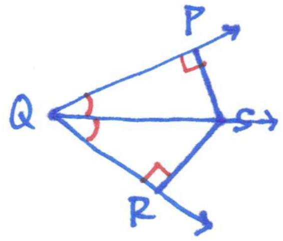


Find length of $\overline{JM}$.

$$9x-18 = 3x$$
$$-3x+18 \quad -3x+18$$
$$\frac{6x}{6} = \frac{18}{6}$$
$$x = 3$$

$$\overline{JM} = 9(3)-18$$
$$\boxed{\overline{JM} = 9}$$

* **Angle Bisector Theorem:** If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

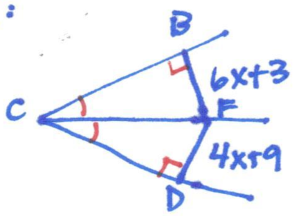


ex: If $\overrightarrow{QS}$ bisects $\angle PQR$ & $\overline{SP} \perp \overrightarrow{QP}$ & $\overline{SR} \perp \overrightarrow{QR}$, then $SP = SR$

* **Converse of Angle Bisector Theorem:** If $\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$, and $SP = SR$, then $\overrightarrow{QS}$ bisects $\angle PQR$

Got it?

ex:

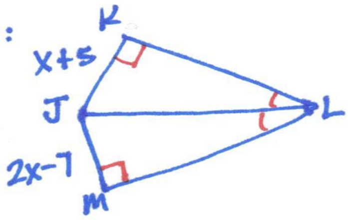


Find length of $\overline{FB}$.

$$\cancel{6x}+\cancel{3}=\cancel{4x}+9$$
$$-4x-\cancel{3}\quad -4x-3$$
$$2x=6$$
$$x=3 \rightarrow \overline{FB}=6(3)+3=\boxed{21}$$

#6

ex:



Find x, $\overline{KJ}$, $\overline{JM}$.

$$\cancel{x}+5=2x-\cancel{7}$$
$$-x+7\quad -x+7$$
$$\boxed{12=x}$$

$$\boxed{\begin{array}{l}\overline{KJ}=12+5=17\\ \overline{JM}=2(12)-7=17\end{array}}$$

Indirect Proof (Section 8-5)

* Writing an Indirect Proof: ① State as a temporary assumption the opposite (negation) of what you want to prove.

② Show that this temp. assumption leads to a contradiction.

③ Conclude that the temp. assumption must be false & what you want to prove must be true.

#1
ex: At least one angle is obtuse.

Assume temp. that no $\angle$ is obtuse.

#2
ex: $m\angle 2 > 90$

Assume temp. that $m\angle 2 \leq 90$

* To write an indirect proof, you must be able to identify a contradiction.

#3
ex: I: Each of the 2 items Val bought cost more than \$10.

II: Val spent \$34 for the 2 items.

III: Neither of the 2 items that Val bought cost more than \$15.

Contradiction: II & III

#4
ex: In right $\triangle ABC$, $m\angle A = 60$. → I

In right $\triangle ABC$, $\angle A \cong \angle C$ → II

In right $\triangle ABC$, $m\angle B = 90$. → III

Contradiction: I & II.

#5
ex: Given: $7(x+y) = 70$ and $x \neq 4$

Prove: $y \neq 6$

① Assume temp. $y = 6$, then

$7(x+6) = 70$

$x + 6 = 10$

$x = 4$ but this ② contradicts $x \neq 4$

③ $\therefore y \neq 6$.

#6

ex: Given: ΔLMN

Prove: ΔLMN has at most one right angle.

① Assume temp. that ΔLMN has more than one right angle. ($\angle M$ & $\angle N$ are $90°$)

$m\angle M + m\angle N = 180°$ This would make $m\angle L = 0$.

② This means there is no ΔLMN which contradicts the given so the temp. assumption must be false. $\therefore$ ΔLMN ③ has at most one right angle.